

# Electron-Electron Interaction in Disordered Mesoscopic Systems: Weak Localization and Mesoscopic Fluctuations of Polarizability and Capacitance

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The weak localization correction and the mesoscopic fluctuations of the polarizability and the capacitance of a small disordered sample are studied systematically in 2D and 3D geometries. While the grand canonical ensemble calculation gives the positive magnetopolarizability, in the canonical ensemble (appropriate for isolated samples) the sign of the effect is reversed. The magnitude of mesoscopic fluctuations for a single sample exceeds considerably the value of the weak localization correction.

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## I. INTRODUCTION

The phenomena of weak localization (WL) and mesoscopic fluctuations in disordered systems have been intensively studied during the last fifteen years, mainly in connection with transport properties of these systems [1,2]. For these phenomena the role of the electron-electron interaction is just in setting the length scale  $l_\phi$  (phase breaking length), below which the electron wave function preserves its phase coherence. Systems with a size  $L$  less than  $l_\phi$  are called mesoscopic systems.

In this paper, we consider quantum corrections and mesoscopic fluctuations of the two other characteristics of a mesoscopic system, where the electron-electron interaction is essential: polarizability and capacitance. The former quantity can be measured by putting a sample into a capacitor, while the latter one determines the charging energy which shows up in the Coulomb blockade experiments [3].

The first quantum calculation of the polarizability of a small metallic particle was obtained in a seminal paper by Gor'kov and Eliashberg (GE) [4]. It relied on the following two assumptions concerning statistical properties of energy levels and eigenfunctions in disordered systems:

1. The single-particle energy spectrum exhibits the same statistics as the eigenvalue spectrum of random matrices from the Gaussian Ensemble of the corresponding symmetry;
2. Exact single-particle eigenfunctions  $\psi_k(\mathbf{r})$  and  $\psi_l(\mathbf{r})$ , which are close enough in energy, are correlated in the following way:

$$V^2 \langle \psi_k^*(\mathbf{r}) \psi_l(\mathbf{r}) \psi_k(\mathbf{r}') \psi_l^*(\mathbf{r}') \rangle_{\epsilon, \omega} = \Pi_D(\mathbf{r}, \mathbf{r}'). \quad (1)$$

Here the average is defined as

$$\begin{aligned} \langle \psi_k^*(\mathbf{r}) \psi_l(\mathbf{r}) \psi_k(\mathbf{r}') \psi_l^*(\mathbf{r}') \rangle_{\epsilon, \omega} \equiv \\ \frac{\langle \sum_{k \neq l} \psi_k^*(\mathbf{r}) \psi_l(\mathbf{r}) \psi_k(\mathbf{r}') \psi_l^*(\mathbf{r}') \delta(\epsilon - \epsilon_k) \delta(\epsilon + \omega - \epsilon_l) \rangle}{\langle \sum_{k \neq l} \delta(\epsilon - \epsilon_k) \delta(\epsilon + \omega - \epsilon_l) \rangle}, \end{aligned} \quad (2)$$

$V$  is the sample volume, and the diffusion propagator  $\Pi_D$  is a solution to the diffusion equation,

$$-D \nabla^2 \Pi_D(\mathbf{r}, \mathbf{r}') = (\pi\nu)^{-1} [\delta(\mathbf{r} - \mathbf{r}') - V^{-1}], \quad (3)$$

with the boundary conditions  $\nabla_{\mathbf{n}} \Pi_D = 0$ . The first of these conjectures was proven by Efetov [5], and the second by the authors [6]. More precisely, it was shown in [6] that for the energy difference much less than the Thouless energy,  $\omega \ll E_c$ ,

$$\begin{aligned} V^2 \langle \psi_k^*(\mathbf{r}) \psi_l(\mathbf{r}) \psi_k(\mathbf{r}') \psi_l^*(\mathbf{r}') \rangle_{\epsilon, \omega} \\ = k_d(\mathbf{r} - \mathbf{r}') + \Pi_D(\mathbf{r}, \mathbf{r}'), \end{aligned} \quad (4)$$

where  $k_d(\mathbf{r}) = (\pi\nu)^{-2} \langle \text{Im} G^R(\mathbf{r}) \rangle^2$  is a short-range function ( $G^R(\mathbf{r})$  being the retarded Green's function) explicitly given by

$$k_d(\mathbf{r}) = \exp(-r/l) \begin{cases} J_0^2(p_F r), & 2D \\ (p_F r)^{-2} \sin^2 p_F r, & 3D \end{cases}. \quad (5)$$

The short-range part  $k_d(\mathbf{r} - \mathbf{r}')$  of the correlation function (4) was not taken into account by Gor'kov and Eliashberg, but it would give only a small correction to their result [7].

Based on these conjectures, GE concluded that the polarizability for very low frequencies  $\omega \ll \Delta$  ( $\Delta = (\nu V)^{-1}$  is the mean single-particle level spacing;  $\nu$  is the density of states per spin) is enhanced in comparison with the classical value  $\alpha_0 \sim V$ , the enhancement factor being of order  $(\kappa R)^2$ , where

$$\kappa = \begin{cases} (8\pi e^2 \nu)^{1/2}, & 3D \\ 4\pi e^2 \nu, & 2D \end{cases} \quad (6)$$

is the inverse screening radius. Although the original paper [4] gave a new insight into the field (called later mesoscopic physics), and had a substantial impact on

the further development of the condensed matter physics, this result for the polarizability is incorrect for the following reason. The paper by Gor'kov and Eliashberg does not take into account the effects of screening: they calculate the polarizability in response to the local field rather than to the external one [8]. As was found in Refs. [9,10] (see also [11]), the screening restores the classical value of the polarizability, thus reducing the quantum effects to a relatively small correction. Evaluation of this correction was recently attempted by Efetov [12], who combined the non-perturbative calculation of the polarization function [6] with the electron-electron interactions taken into account in the RPA approximation. Since the value of the quantum correction depends on the presence (or absence) of the time reversal symmetry, it was denoted by Efetov as a “weak localization correction to polarizability”; we are following this terminology in the present article. However, he estimated incorrectly the contribution of the short-range term in Eq.(4), which made him to conclude that the weak localization correction is dominated by this term. As we show below, this is not the case if the system size exceeds considerably the mean free path. More recently, Noat, Reulet, and Bouchiat [13] presented a perturbative calculation of the weak localization correction to the polarizability in a particular geometry of a narrow 2D ring. They considered both canonical (CE) and grand-canonical (GCE) ensembles, and concluded that the correction to the polarizability is parametrically suppressed in the CE. While essentially confirming their GCE result, we disagree with the above statement concerning the CE. We show below that the effect in the CE is of the same (up to a coefficient of order one) magnitude than the GCE one, but has an opposite sign.

As was realized by Berkovits and Altshuler [14], fluctuations in the polarization function lead to mesoscopic fluctuations of the polarizability of the sample. They considered a specific thin film geometry and identified the four-diffuson diagrams giving the leading contribution to the fluctuations. We will follow their approach when studying the polarizability fluctuations in 2D and 3D geometries.

Along with the polarizability, we consider another quantity characterizing a mesoscopic system, the capacitance  $C$ . It determines the charging energy  $e^2/C$ , which manifests itself in the  $I - V$  characteristics of a quantum dot in the Coulomb blockade regime. In particular, the charging energy represents the main contribution to the threshold voltage in the excitation spectra and to the distance between adjacent conductance peaks in the addition spectra. Statistical properties of the Coulomb blockade  $I - V$  characteristics are attracting a great research interest now [15–21], which motivated us to consider the WL correction and the mesoscopic fluctuations of the charging energy. In addition, the capacitance determines the low-frequency behavior of the impedance of mesoscopic systems [22,23].

Therefore, the purpose of the present paper is to study

*systematically* the WL effects and mesoscopic fluctuations of the polarizability and capacitance in 2D and 3D geometry. Where it is necessary, we refine results of previous research. We show that the polarizability and the capacitance can be treated on the same physical grounds. We will also find a simple relation between the magnitude of the WL correction and that of the mesoscopic fluctuations. The electron-electron interaction is taken into account in the RPA approximation, which works for  $\kappa < p_F$ ,  $p_F$  being the Fermi momentum. We consider the case of low temperature  $T \ll \Delta$  (thus setting  $T = 0$  in all formulas), and study both grand-canonical and canonical ensembles.

## II. WEAK LOCALIZATION CORRECTION TO THE POLARIZABILITY OF SMALL PARTICLES

We consider an isolated disordered metallic particle  $\Omega$  (3D or 2D) placed into a uniform external frequency-dependent electric field  $\mathbf{E}(\omega)$ . We assume that the system is diffusive,  $l \ll L$ , where  $l$  and  $L$  are a mean free path and a typical size of a particle, respectively. In the RPA approximation the potential distribution  $\Phi(\mathbf{r})$  and the electron density  $\rho(\mathbf{r})$  in the particle obey the Poisson equation ( $e = -|e|$  being the electron charge),

$$\Delta\Phi(\mathbf{r}) = -4\pi e\rho(\mathbf{r})\theta_\Omega(\mathbf{r}) \begin{cases} 1, & 3D \\ \delta(z), & 2D \end{cases},$$

$$\theta_\Omega(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in \Omega \\ 0, & \text{otherwise} \end{cases}, \quad (7)$$

in combination with the equation

$$\rho(\mathbf{r}) = -2e \int_\Omega \Pi(\mathbf{r}, \mathbf{r}') \Phi(\mathbf{r}') d\mathbf{r}'. \quad (8)$$

In 2D we use the following convention throughout the paper:  $\mathbf{r} = (x, y)$  denotes the coordinates in plain, and  $z$  the transverse coordinate. The Laplacian  $\Delta$  is always a three-dimensional operator,  $\Delta \equiv \Delta_3 = \Delta_2 + \partial_z^2$ . Furthermore,  $\Pi$  is the polarization function (per spin), which can be readily expressed through the Matsubara Green's functions,

$$\Pi(\mathbf{r}, \mathbf{r}', \omega) = -T \sum_{\epsilon_m} \langle G(\mathbf{r}, \mathbf{r}', i\epsilon_m + i\omega_n) \times G(\mathbf{r}', \mathbf{r}, i\epsilon_m) \rangle |_{i\omega_n \rightarrow \omega + i0}, \quad (9)$$

or, in terms of the retarded and advanced Green's functions  $G^{R,A}(\mathbf{r}, \mathbf{r}', \epsilon)$ ,

$$\begin{aligned} & \Pi(\mathbf{r}, \mathbf{r}', \omega) \\ &= \frac{-i}{2\pi} \int d\epsilon \{ n_F(\epsilon) \langle G^R(\mathbf{r}, \mathbf{r}', \epsilon + \omega) G^R(\mathbf{r}', \mathbf{r}, \epsilon) \rangle - \\ & \quad - n_F(\epsilon + \omega) \langle G^A(\mathbf{r}, \mathbf{r}', \epsilon + \omega) G^A(\mathbf{r}', \mathbf{r}, \epsilon) \rangle + \\ & \quad + [n_F(\epsilon + \omega) - n_F(\epsilon)] \langle G^R(\mathbf{r}, \mathbf{r}', \epsilon + \omega) G^A(\mathbf{r}', \mathbf{r}, \epsilon) \rangle \} \\ &\equiv \Pi^{RR}(\mathbf{r}, \mathbf{r}', \omega) + \Pi^{AA}(\mathbf{r}, \mathbf{r}', \omega) + \Pi^{RA}(\mathbf{r}, \mathbf{r}', \omega). \end{aligned} \quad (10)$$

While  $\Pi^{RR}$  and  $\Pi^{AA}$  can be calculated in the usual impurity perturbation theory, the evaluation of  $\Pi^{RA}$  for low frequencies,  $\omega \lesssim \Delta$ , requires a non-perturbative treatment. This was done in Ref. [6] in the framework of the supersymmetric sigma-model approach, and we present only the results here:

$$\begin{aligned} & \Pi^{RR}(\mathbf{r}, \mathbf{r}', \omega) + \Pi^{AA}(\mathbf{r}, \mathbf{r}', \omega) \\ &= \frac{1}{\pi} \int_{-\infty}^0 d\epsilon \text{Im} \langle G^R(\mathbf{r}, \mathbf{r}', \epsilon) \rangle^2 \\ &+ \frac{\omega}{2\pi i} [\langle \text{Re} G^R(\mathbf{r}, \mathbf{r}', 0) \rangle^2 - \langle \text{Im} G^R(\mathbf{r}, \mathbf{r}', 0) \rangle^2]; \\ \Pi^{RA}(\mathbf{r}, \mathbf{r}', \omega) &= -\frac{\nu}{V} - \frac{\omega}{2\pi i} \{ \langle \text{Re} G^R(\mathbf{r}, \mathbf{r}', 0) \rangle^2 \\ &+ S(\omega) \langle \text{Im} G^R(\mathbf{r}, \mathbf{r}', 0) \rangle^2 + (\pi\nu)^2 (1 + S(\omega)) \Pi_D(\mathbf{r}, \mathbf{r}') \}. \end{aligned} \quad (11)$$

Here  $S(\omega)$  is a correlation function of the zero-dimensional sigma-model,  $S(\omega) = -\langle Q_{bb}^{11} Q_{bb}^{22} \rangle$  (in notations of Ref. [6]), given explicitly by

$$S(\omega) = 1 + \frac{2i\Delta^2}{\pi^2\omega^2} \exp\left(\frac{\pi i\omega}{\Delta}\right) \sin \frac{\pi\omega}{\Delta}. \quad (12)$$

It is related to the two-level correlation function  $R_2(\omega)$  as  $R_2(\omega) = \text{Re}[1 + S(\omega)]/2$ . Now we decompose the polarization function into frequency-independent and frequency-dependent parts:

$$\begin{aligned} \Pi(\mathbf{r}, \mathbf{r}', \omega) &= \Pi_0(\mathbf{r}, \mathbf{r}') + \Pi_1(\mathbf{r}, \mathbf{r}', \omega), \\ \Pi_0(\mathbf{r}, \mathbf{r}') &= \frac{1}{\pi} \text{Im} \int_{-\infty}^0 d\epsilon \langle G^R(\mathbf{r}, \mathbf{r}', \epsilon) \rangle^2 - \frac{\nu}{V}, \\ \Pi_1(\mathbf{r}, \mathbf{r}', \omega) &= -\frac{\omega}{2\pi i} (1 + S(\omega)) \\ &\times [\langle \text{Im} G^R(\mathbf{r}, \mathbf{r}', 0) \rangle^2 + (\pi\nu)^2 \Pi_D(\mathbf{r}, \mathbf{r}')] \\ &= \frac{\nu}{V} A(\omega) [k_d(\mathbf{r} - \mathbf{r}') + \Pi_D(\mathbf{r}, \mathbf{r}')] , \end{aligned} \quad (14)$$

where we defined  $A(\omega) = (i\pi\omega/2\Delta)(1 + S(\omega))$  and introduced the function  $k_d(\mathbf{r} - \mathbf{r}') = \langle \text{Im} G^R(\mathbf{r}, \mathbf{r}', 0) \rangle^2$  given explicitly by Eq.(5). Note that the formulas (11), (12), (14) are written for the case of the unitary ensemble (broken time-reversal symmetry due to the presence of a strong enough magnetic field), which was considered in Ref. [6]. Generalization to the orthogonal ensemble (unbroken time-reversal symmetry) is straightforward and results in the following modification of the factor  $A(\omega)$  in Eq.(14) for  $\Pi_1$ :

$$A(\omega) = \frac{i\pi\omega}{2\Delta} (1 + S(\omega)) - 1 , \quad (15)$$

where now

$$S(\omega) = 1 + \frac{2ie^{is} \sin s}{s^2} + 2i \frac{d}{ds} \left( \frac{\sin s}{s} \right) \int_1^\infty \frac{e^{ist}}{t} dt \quad (16)$$

and  $s = \pi\omega/\Delta$ . In the limit of low frequency,  $\omega \ll \Delta$ , the factor  $A(\omega)$  is equal to

$$A(\omega \ll \Delta) = -\frac{2}{\beta} \equiv \begin{cases} -1, & \text{unitary} \\ -2, & \text{orthogonal} \end{cases} , \quad (17)$$

where  $\beta$  is the usual parameter equal to 1 (2) for the orthogonal (resp. unitary) ensemble.

Now we turn to the calculation of the dipole moment. The general expressions were obtained in Ref. [12]; we present here the derivation for the sake of completeness. We consider the frequency-dependent part  $\Pi_1$  as a perturbation and expand functions  $\rho(\mathbf{r})$ ,  $\Phi(\mathbf{r})$ , and the dipole moment,

$$\mathbf{d} = e \int \mathbf{r} \rho(\mathbf{r}) d\mathbf{r},$$

with respect to  $\Pi_1$ . In the zero-order approximation we obtain

$$\mathbf{d}_0 = -2e \int_{\Omega} \mathbf{r} \Pi_0(\mathbf{r}, \mathbf{r}') \Phi_0(\mathbf{r}') d\mathbf{r} d\mathbf{r}', \quad (18)$$

where the potential  $\Phi_0(\mathbf{r})$  satisfies Eqs. (7), (8) with  $\Pi$  replaced by  $\Pi_0$ .

FIG. 1. The electrostatic potential and the electron density in the RPA approximation. The external dashed line is the bare potential of the electric field  $-\mathbf{E}\mathbf{r}$ , the loops with 0 and 1 denote the contributions  $\Pi_0$  and  $\Pi_1$  to the polarization function, respectively. The wavy line is the Coulomb interaction.

It is easy to check that the first term in the expression (13) for  $\Pi_0(\mathbf{r}, \mathbf{r}')$  gives  $\nu$  after integration over one of the coordinates, the integral being dominated by the distances of order of the Fermi wave length,  $|\mathbf{r} - \mathbf{r}'| \sim \lambda_F$ .

Assuming the screening length (which sets the scale for the variation of the potential  $\Phi(\mathbf{r})$ ) to be larger than  $\lambda_F$ , we can thus replace this term by a delta-function:

$$\Pi_0(\mathbf{r}, \mathbf{r}') = \nu [\delta(\mathbf{r} - \mathbf{r}') - V^{-1}]. \quad (19)$$

This approximation for the polarization function  $\Pi_0$  leads to the simple relation between the potential  $\Phi_0(\mathbf{r})$  and the excess density  $\rho_0(\mathbf{r})$  (Thomas-Fermi approximation)

$$\Phi_0(\mathbf{r}) = -(2e\nu)^{-1} \rho_0(\mathbf{r}), \quad (20)$$

and consequently to the following equation for the potential  $\Phi_0(\mathbf{r})$ ,

$$\Delta\Phi_0(\mathbf{r}) = \begin{cases} \kappa^2\Phi_0(\mathbf{r})\theta_\Omega(\mathbf{r}), & 3D \\ \kappa\delta(z)\Phi_0(\mathbf{r})\theta_\Omega(\mathbf{r}), & 2D \end{cases}, \quad (21)$$

with  $\kappa$  defined in Eq.(6) and the boundary condition  $\Phi_0(r \rightarrow \infty) = -\mathbf{E}\mathbf{r} + \text{const.}$  In Eqs.(20), (21) we have chosen the arbitrary additive constant in the definition of  $\Phi_0$  in such a way that  $\int_\Omega \Phi_0(\mathbf{r})d\mathbf{r} = 0$ .

In the following we consider particular geometries of a 3D spherical sample of a radius  $R$  (to be referred as 3D), a circle of a radius  $R$  in the in-plane electric field (2D), and a quasi-two-dimensional sample of a thickness  $h$  ( $0 < z < h$ ) and an area  $S$  in the field directed along the  $z$ -axis, i.e. transverse to the sample (Q2D). The polarizability tensor  $\alpha_{ij}$  is generally defined as  $\mathbf{d}(\omega) = \hat{\alpha}(\omega)\mathbf{E}(\omega)$ . Eqs. (18) and (21) yield the classical polarizability [25] equal in the limit  $\kappa R \ll 1$ ,  $\kappa h \ll 1$  to (for all the geometries under consideration the dipole moment is directed along the field, and the tensor is reduced to a scalar),

$$\alpha_0 \simeq \begin{cases} R^3, & 3D \\ Sh/4\pi, & \text{Q2D} \\ (4/3\pi)R^3, & 2D \end{cases}. \quad (22)$$

Now we turn to the corrections due to the function  $\Pi_1$ . One obtains (Fig.1)

$$\mathbf{d}_1\mathbf{E} = 2e^2 \int d\mathbf{r}d\mathbf{r}' \Phi_0(\mathbf{r})\Pi_1(\mathbf{r}, \mathbf{r}')\Phi_0(\mathbf{r}'), \quad (23)$$

in full accordance with Ref. [12]. Note that Eq. (23) depends explicitly on the symmetry of the system with respect to the time reversal, and therefore constitutes the WL correction to the polarizability. As follows from Eq. (14), this correction consists of two contributions. The first one (to be referred as  $\alpha_{1S}$ ) is due to the short-range contribution to the polarization function (the first term in the brackets, Eq. (14)), and the second one ( $\alpha_{1D}$ ) is due to the diffusion contribution  $\Pi_D$ .

To evaluate the second term, we use the expansion of  $\Pi_D$  in the eigenfunctions of the Laplace operator  $\phi_\alpha(\mathbf{r})$  with the boundary conditions  $\nabla\phi_\alpha = 0$  and the corresponding eigenvalues  $-\epsilon_\alpha$ ,

$$\Pi_D(\mathbf{r}, \mathbf{r}') = (\pi D\nu)^{-1} \sum_{\alpha \neq 0} \epsilon_\alpha^{-1} \phi_\alpha(\mathbf{r})\phi_\alpha(\mathbf{r}'). \quad (24)$$

For the purposes of estimate, one can use the following expressions valid for  $l \ll |\mathbf{r} - \mathbf{r}'| \ll L$  ( $L$  is a typical size of the system),

$$\Pi_D(\mathbf{r}, \mathbf{r}') \approx \begin{cases} (2\pi^2\nu D)^{-1} \ln[L/|\mathbf{r} - \mathbf{r}'|], & 2D \\ (4\pi^2\nu D|\mathbf{r} - \mathbf{r}'|)^{-1}, & 3D \end{cases}. \quad (25)$$

Now we evaluate and compare both contributions  $\alpha_{1S}$  and  $\alpha_{1D}$  in 3D and 2D systems. Since the structure of the potential  $\Phi_0$  is different in 3D and 2D cases, these should be treated separately.

### A. 3D geometry

For any 3D geometry with  $\kappa L \gg 1$  the expression for  $\Phi_0(\mathbf{r})$  can be written in the form

$$\Phi_0(\mathbf{r}) = \frac{E}{\kappa} \varphi(\mathbf{r}_\parallel) \exp(-\kappa r_\perp), \quad (26)$$

with  $\varphi$  being some function of magnitude unity. We have introduced a transverse coordinate  $r_\perp$  ( $r$  for the sphere,  $z$  in the case of a disk), and the vector  $\mathbf{r}_\parallel$  of coordinates along the surface of the sample. Note that according to Eq.(20),

$$\varphi(\mathbf{r}_\parallel) = -4\pi e\sigma_0(\mathbf{r}_\parallel)/E, \quad (27)$$

where  $e\sigma_0(\mathbf{r}_\parallel)$  is the charge density on a surface of an ideal conductor induced by the electric field  $E$ . It can be found by the methods of the classical electrostatics [25]. In the integral for  $\alpha_{1S}$ ,

$$\alpha_{1S} = \frac{2e^2\nu A(\omega)}{VE^2} \times \int d^3\mathbf{r}d^3\mathbf{r}' \Phi_0(\mathbf{r})k_d(|\mathbf{r} - \mathbf{r}'|)\Phi_0(\mathbf{r}'), \quad (28)$$

both points  $\mathbf{r}$  and  $\mathbf{r}'$  lie in fact in the layer of thickness  $\kappa^{-1} \ll l$  near the surface of the sample. One can then integrate over the transverse coordinates and reduce the remaining double surface integral to the integral over one coordinate only. We obtain

$$\alpha_{1S} \simeq \frac{1}{Vp_F^2\kappa^2} A(\omega) \ln(\kappa l) \int d^2\mathbf{r}_\parallel \varphi^2(\mathbf{r}_\parallel) \sim \frac{1}{Lp_F^2\kappa^2} A(\omega) \ln(\kappa l). \quad (29)$$

In Ref. [12] the kernel in Eq. (28) was incorrectly replaced by a  $\delta$ -function, which led to an overestimate of the contribution  $\alpha_{1S}$  by a factor of  $\kappa l(\ln(\kappa l))^{-1} \gg 1$ .

On the other hand, for the term due to the diffusion,

$$\alpha_{1D} = \frac{2e^2\nu}{VE^2} A(\omega) \int d\mathbf{r}d\mathbf{r}' \Phi_0(\mathbf{r})\Pi_D(\mathbf{r}, \mathbf{r}') \times \Phi_0(\mathbf{r}'), \quad (30)$$

we obtain, using the estimate (25),

$$\alpha_{1D} \sim \frac{1}{\nu D \kappa^2} A(\omega) \sim \alpha_{1S} \frac{L}{l \ln(\kappa l)}. \quad (31)$$

We see that in a diffusive system of a size  $L \gg l \ln(\kappa l)$ , the diffusion contribution  $\alpha_{1D}$  dominates, in contrast to the conclusion of Ref. [12]. At the same time, if the sample size  $L$  is comparable to the mean free path  $l$  (which happens e.g. in ballistic systems with surface scattering), the short-range contribution is parametrically of the same order (in fact, even larger by a logarithmic factor) as the diffusive one. As expected [9,10,12], the WL correction is small in comparison with the classical polarizability  $\alpha_0$ ,

$$\alpha_1/\alpha_0 \sim \frac{1}{g(\kappa L)^2} A(\omega), \quad (32)$$

$g \sim 2\pi\nu DL$  being the dimensionless conductance. The calculation of the numerical coefficient for the WL correction to the polarizability requires the exact expansion (24). For the particular spherical geometry the potential  $\Phi_0$  has a form

$$\Phi_0(\mathbf{r}) = -\frac{3ER}{\pi\kappa r} \exp(-\kappa(R-r)) \cos\theta, \quad R-r \ll R, \quad (33)$$

and we obtain

$$\alpha_1 = \frac{1.36}{(p_F\kappa)^2 l} A(\omega). \quad (34)$$

According to Eq.(17), the WL correction to the polarizability is negative. The value of the correction in the presence of strong magnetic field (unitary symmetry) is smaller (twice as small for zero frequency) as without the latter (orthogonal symmetry). The experimentally measured magnetopolarizability  $\alpha_B$ , defined as

$$\alpha_B = \alpha(B) - \alpha(0), \quad (35)$$

is therefore positive, in agreement with Ref. [12].

### B. Quasi-2D geometry (transverse field).

We consider now a quasi-two-dimensional sample of a thickness  $h \gg \kappa^{-1}$  and an area  $S \gg h^2$  with the electric field directed transverse to the sample plane. Then Eq.(26) for the potential reduces to

$$\Phi_0(\mathbf{r}) = \frac{E}{\kappa} (-e^{-\kappa z} + e^{-\kappa(h-z)}) \quad (36)$$

If the sample is relatively thick,  $h \gg l$ , the same consideration as for the case of a spherical shape yields

$$\alpha_{1S} = A(\omega) \frac{\ln(\kappa l)}{h(p_F\kappa)^2}, \quad \alpha_{1D} = \frac{3}{2} A(\omega) \frac{1}{l(p_F\kappa)^2}, \quad (37)$$

and the diffusion term dominates for  $h > l \ln(\kappa l)$ . In the opposite case of a thin sample the short-range contribution is the leading one. In particular, for  $h < l$  we find

$$\alpha_1 \simeq \alpha_{1S} = A(\omega) \frac{\ln(\kappa h)}{h(p_F\kappa)^2} \quad (38)$$

As is seen from the above formulas, the relative magnitude of the weak localization correction is rather low for both 3D and quasi-2D (with the field direction normal to the plane) geometries, so that the experimental observation of the effect in these cases may be problematic. The effect is much more pronounced in the 2D case, which we consider below.

### C. 2D geometry (in-plane field).

In contrast to the 3D case, the potential  $\Phi_0$  in the case of a 2D sample in the in-plane electric field is a smooth function of coordinates, with the characteristic scale set by the sample size  $R$ . Therefore the kernel in the integral (28), which has a support of order  $l$ , can be replaced by a  $\delta$ -function,

$$(p_F r)^{-1} \exp(-r/l) \approx 2\pi l p_F^{-1} \delta(\mathbf{r}).$$

This gives an estimate

$$\alpha_{1S} \sim l(p_F\kappa)^{-1} A(\omega) \ln(R/l).$$

On the other hand, for the diffusive term (30) the estimate (25) implies

$$\alpha_{1D} \sim R^2(\kappa g)^{-1} A(\omega) \sim \alpha_{1S} (R/l)^2 (\ln(R/l))^{-1}.$$

Similarly to the 3D case, the diffusion term  $\alpha_{1S}$  dominates for  $R \gg l$ . The relative magnitude of the quantum correction can thus be estimated as

$$\alpha_1/\alpha_0 \sim 1/g\kappa R,$$

with  $g = 2\pi\nu D = k_F l/2$ .

For the particular case of a circular geometry, the potential  $\Phi_0$  is given in the polar coordinates  $(r, \theta)$  by [25]

$$\Phi_0(\mathbf{r}) = -2E(\pi\kappa)^{-1} r \cos\theta (R^2 - r^2)^{-1/2} \quad (39)$$

An exact calculation gives the value of the quantum correction

$$\alpha_1(\omega) = 1.53 R^2 (\kappa p_F l)^{-1} A(\omega),$$

and the relative magnitude of the correction is

$$\alpha_1/\alpha_0 = 3.6 \times \frac{1}{\kappa p_F l R} A(\omega)$$

In a recent paper, Noat, Reulet and Bouchiat (NRB) [13] proposed a geometry of a narrow 2D ring (radius

$R$ , width  $W \ll R$ ) as more favorable for observation of the effect. In the in-plane electric field the ring becomes polarized with the one-dimensional (i.e. integrated over the ring cross-section) charge density

$$\rho(\theta) = \frac{ER}{e \ln(R/W)} \cos \theta,$$

and the classical polarizability given by

$$\alpha_0 = \frac{\pi R^3}{\ln(R/W)}.$$

Calculating the quantum correction, we find again that for a diffusive ring  $R \gg l$ , the contribution  $\alpha_{1D}$  dominates and gives

$$\alpha_1 = \frac{R^4}{\nu DW^2 \kappa \ln^2(R/W)} A(\omega). \quad (40)$$

The relative magnitude of the correction is

$$\alpha_1/\alpha_0 = \frac{1}{\pi g W \kappa \ln(R/W)} A(\omega), \quad (41)$$

where  $g$  is now the quasi-one-dimensional conductance  $g = \nu DW/R$ . These results for the ring geometry are by and large in agreement with those found by NRB [13]. Actually, NRB express the polarizability in terms of the exact eigenfunctions of electrons, conceptually similarly to the original GE calculation, and then perform the impurity averaging using the semi-classical expression for the correlation of the exact single-particle eigenfunctions (see e.g. [1,24]). This calculation yields correct results for the following reasons. First, the short-ranged terms in Eq. (1), omitted in this calculation, turn out to be unimportant for the WL correction to the polarizability. Then, the *exact* expression for the long-ranged (diffusive) part of the eigenfunction correlator (1) coincides with the semi-classical result even for  $\omega \ll \Delta$ , where the latter generally is not expected to be true. This has been proved and discussed previously by the authors in Ref. [6].

Similarly, we can consider a quasi-one-dimensional strip of width  $W$  and length  $L \gg W$  oriented along the electric field direction (which we choose to be the  $z$ -axis). Again, the sample polarization is described by the one-dimensional charge density

$$\rho(z) = \frac{Ez}{e \ln(L/W)},$$

yielding the classical polarizability

$$\alpha_0 = \frac{L^3}{12 \ln(L/W)}.$$

The quantum correction is now equal to

$$\alpha_1 \simeq \alpha_{1D} = \frac{\pi L^3}{30 \kappa W g \ln^2(L/\omega)} A(\omega), \quad (42)$$

where  $g = 2\pi\nu DW/L$  is the dimensionless conductance. We obtain

$$\alpha_1/\alpha_0 = \frac{2\pi}{5} \frac{1}{\kappa W g \ln(L/W)} A(\omega). \quad (43)$$

Thus, we have found that the WL correction to the polarizability can be quite appreciable in 2D (circle) and especially in quasi-1D (ring or strip) geometries, which gives a possibility of its experimental observation. These conclusions are in full agreement with those of NRB, Ref. [13].

## D. Canonical ensemble

The results obtained above were derived for the grand canonical ensemble, where the chemical potential is fixed by an external reservoir. In the Appendix A we present the calculations for the canonical ensemble, which is more appropriate for the problem in question [26]. We show (Eq. (62)) that the CE magnetopolarizability differs from the GCE result by the coefficient  $-2.75$ . This means that although the magnitude of the CE effect is the same as in the GCE, the sign is opposite in the CE case: the magnetic field suppresses the polarizability.

## III. WEAK LOCALIZATION EFFECTS IN THE CAPACITANCE

### A. Definitions

A natural definition of the capacitance in an open system is

$$C_\mu = edQ/d\mu, \quad (44)$$

where  $Q$  is the total charge of the system. This capacitance determining the low-frequency transport properties of the system was studied thoroughly by Büttiker and coworkers in Refs. [22,27] (where it was called “electrochemical capacitance”). An explicit calculation [22] yields

$$\frac{e^2}{C_\mu} = \frac{e^2}{C_g} + \frac{1}{V\nu(E_F)}, \quad (45)$$

where  $C_g$  is the geometrical capacitance, determined from the equations of the classical electrostatics [25] with corrections due to the screening effects, and  $\nu(E_F)$  is the density of states at the Fermi level. The average of the second term in the right-hand side of Eq. (45) is the mean level spacing  $\Delta$ , which is usually much less than the average of the first term. However, the fluctuations of the second term are important [23].

Being formally applied to the closed system, Eq. (44) yields infinite fluctuations of the charging energy. Indeed,

in Ref. [19], where an attempt to calculate the fluctuations of the compressibility of a closed system has been made, the integral over energies diverged, and the authors could get a finite result only by cutting it off at energies of order  $\Delta$ .

A proper generalization of the definition (44) for a closed system is its “discrete” version,

$$e^2/C_\mu = \mu(N+1) - \mu(N), \quad (46)$$

where  $\mu(N)$  is the chemical potential of a *closed* system of  $N$  electrons. The quantity (46) has an important physical meaning: it is equal to the spacing between two consecutive peaks in the addition spectrum of a quantum dot in the Coulomb blockade regime. Statistical properties of these spacings were studied experimentally in Refs. [16–18] and theoretically in Ref. [20,21].

Similarly to the case of an open system (Eq.(45)), the peak spacing (46) can be decomposed into two parts: level spacing  $\Delta_N$  and (usually much larger) contribution associated with Coulomb interaction (denoted  $E_1$  in Ref. [20]). The main contribution to the latter (and thus to the Coulomb blockade peak spacing) is given by the charging energy  $E_C$  defined [20] as a constant part of the effective two-particle interaction potential  $U(\mathbf{r}, \mathbf{r}')$  of the electrons in the sample:

$$E_C \equiv e^2/C \equiv V^{-2} \int_{\Omega} d\mathbf{r} d\mathbf{r}' U(\mathbf{r}, \mathbf{r}'). \quad (47)$$

In particular, the WL correction to the charging energy calculated below is the dominating term for the WL in the Coulomb blockade peak spacing, and can be, in principle, measured as the magnetic field dependence of the peak spacing. In contrast, fluctuations of the charging energy do not give the dominant contribution to the fluctuations of the peak spacing, see Sections IV, V, and Ref. [20].

$$\begin{aligned}
\mathbf{U}_1 &= \sim \left( \mathbf{1} - \text{loop}(0) \right) \\
&+ \text{loop}(0) \sim \text{loop}(0) \sim \dots \Big) \\
&\times \text{loop}(1) \left( \mathbf{1} - \sim \text{loop}(0) \right) \\
&+ \sim \text{loop}(0) \sim \text{loop}(0) \sim \dots \Big) \sim \\
&= \sim \left( \mathbf{1} + \text{loop}(0) \right)^{-1} \\
&\times \text{loop}(1) \left( \mathbf{1} + \sim \text{loop}(0) \right)^{-1} \sim
\end{aligned}$$

FIG. 2. RPA approximation for the two-particle potential  $U$ . The potential  $U_0$  is given by the same sequence of diagrams as the potential  $\Phi_0$  in Fig. 1 provided the external dashed line is replaced by the Coulomb interaction (the wavy line).

### B. Weak localization effects

The potential  $U(\mathbf{r}, \mathbf{r}')$  can be found in the RPA approximation [20] (see Fig. 2). As in the case of the polarizability, it is convenient to split the polarization function into two parts (19) and (14). Assuming the low frequency limit  $\omega \ll \Delta$ , we replace the function  $A(\omega)$  by its zero-frequency value, Eq.(17). In the zeroth order in  $\Pi_1$  one should solve an equation

$$U_0(\mathbf{r}, \mathbf{r}') = V_0(\mathbf{r} - \mathbf{r}') - 2 \int_{\Omega} d\mathbf{r}_1 d\mathbf{r}_2 V_0(\mathbf{r} - \mathbf{r}_1) \\ \times \Pi_0(\mathbf{r}_1, \mathbf{r}_2) U_0(\mathbf{r}_2, \mathbf{r}'), \quad V_0(r) = e^2/r. \quad (48)$$

This equation was solved for an arbitrary closed system in Ref. [20]. The result is

$$U_0(\mathbf{r}, \mathbf{r}') = \bar{U} + \tilde{\Phi}_0(\mathbf{r}) + \tilde{\Phi}_0(\mathbf{r}') + U_\kappa(\mathbf{r}, \mathbf{r}'). \quad (49)$$

Here  $\bar{U} \equiv (e^2/C)_0$  is a constant, corresponding to the charging energy calculated in the Thomas-Fermi approximation;  $U_\kappa$  is the usual screened Coulomb potential, shifted by a constant so that  $\int d\mathbf{r} U_\kappa(\mathbf{r}, \mathbf{r}') = 0$ , while  $\tilde{\Phi}_0$  is the contribution due to the excess positive charge, moved towards the boundary of the system after an extra electron is added to the system. For the sphere (3D) and circle (2D) geometries this potential has an explicit form [20]

$$\tilde{\Phi}_0(\mathbf{r}) = \text{const} - \begin{cases} e^2(\kappa R^2)^{-1} \exp(-\kappa(R-r)), & 3D \\ e^2(2\kappa R)^{-1}(R^2-r^2)^{-1/2}, & 2D \end{cases},$$

where the constant is chosen in such a way that  $\int d\mathbf{r} \tilde{\Phi}_0(\mathbf{r}) = 0$ .

In the first order in  $\Pi_1$  we obtain (Fig. 2)

$$U_1(\mathbf{r}, \mathbf{r}') = 2 \int d\mathbf{r}_1 d\mathbf{r}_2 U_0(\mathbf{r}, \mathbf{r}_1) \Pi_1(\mathbf{r}_1, \mathbf{r}_2) U_0(\mathbf{r}_2, \mathbf{r}'),$$

and, taking into account that the integral of  $\Pi_1$  over any of the coordinates is zero, we write the corresponding contribution to the charging energy (which constitutes the WL correction) in the form

$$\left(\frac{e^2}{C}\right)_1 = 2 \int d\mathbf{r}_1 d\mathbf{r}_2 \tilde{\Phi}_0(\mathbf{r}_1) \Pi_1(\mathbf{r}_1, \mathbf{r}_2) \tilde{\Phi}_0(\mathbf{r}_2). \quad (50)$$

Due to the structure of the function  $\Pi_1$  (Eq. (8)) there are two contributions to the WL correction: one comes from the short-ranged term, and another one is related to the diffusion. A comparison of these contributions can be carried out exactly in the same way as was done for the polarizability, and it turns out that the diffusive term dominates if the sample size exceeds considerably the mean free path. Thus, we obtain finally

$$\left(\frac{e^2}{C}\right)_1 = -\frac{4\nu}{\beta V} \int d\mathbf{r}_1 d\mathbf{r}_2 \tilde{\Phi}_0(\mathbf{r}_1) \Pi_D(\mathbf{r}_1, \mathbf{r}_2) \tilde{\Phi}_0(\mathbf{r}_2). \quad (51)$$

The calculation for the particular geometries of a sphere (3D) and disk (2D) gives

$$\left(\frac{e^2}{C}\right)_1 = \begin{cases} -1.32\beta^{-1}\tau^{-1}(p_F R)^{-4}, & 3D \\ -0.010\beta^{-1}\tau^{-1}(p_F R)^{-2}, & 2D \end{cases} \quad (52)$$

The small coefficient in front of the 2D expression is an artifact of the specific circle geometry.

The weak localization correction suppresses the charging energy, i.e. enhances the capacitance. The magnetic field suppresses the capacitance. Both in 2D and 3D cases, the WL correction to the charging energy can be estimated as

$$\left(\frac{e^2}{C}\right)_1 \sim \frac{\Delta}{g}. \quad (53)$$

The WL correction can be in principle extracted from the measurements of the magnetic field dependence of the capacitance, though its rather small value may make such a measurement problematic.

As for the polarizability, the CE calculation yields an additional factor  $-2.75$  in Eqs. (51) and (52). Thus, in CE the magnetic field suppresses the charging energy, enhancing the capacitance.

#### IV. MESOSCOPIC FLUCTUATIONS

Mesoscopic fluctuations of the polarizability and the capacitance can be calculated in a similar way. In addition to the average polarization function (13), (14) there is also a random part  $\Pi_r(\mathbf{r}, \mathbf{r}')$  with the zero average, giving rise to the fluctuations of these quantities. Since the integral of  $\Pi_r$  over each coordinate is zero, we immediately arrive to the expressions for the random parts of the static [28] polarizability  $\alpha_r$  and the charging energy  $(e^2/C)_r$  in precisely the same way as Eqs. (23) and (50) were obtained:

$$\alpha_r = \frac{2e^2}{E^2} \int d\mathbf{r} d\mathbf{r}' \Phi_0(\mathbf{r}) \Pi_r(\mathbf{r}, \mathbf{r}') \Phi_0(\mathbf{r}') \quad (54)$$

and

$$\left(\frac{e^2}{C}\right)_r = 2 \int d\mathbf{r} d\mathbf{r}' \tilde{\Phi}_0(\mathbf{r}) \Pi_r(\mathbf{r}, \mathbf{r}') \tilde{\Phi}_0(\mathbf{r}'). \quad (55)$$

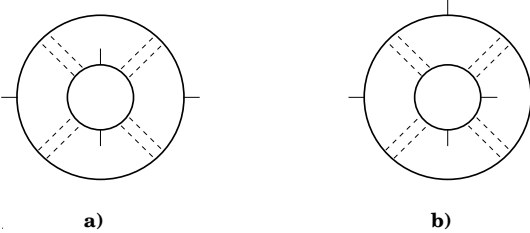


FIG. 3. Diagrams for the fluctuations of the polarization function. The double dashed lines denote the diffusion propagators. The counting factors are 2 (a) and 4 (b) for the unitary symmetry, and 4 (a) and 8 (b) for the orthogonal one.

Thus, the mesoscopic fluctuations of both quantities are determined by the fluctuations of the polarizability. To calculate them, we perform the perturbative calculation. As we will see, in the case of fluctuations the whole range of energies  $\Delta \lesssim \epsilon \lesssim E_c$  contributes, and this fact justifies the perturbative calculation (in contrast to the weak localization correction, which is determined by low energies). Following Berkovits and Altshuler [14], we identify the four-diffusion diagrams (Fig. 3) as giving the main contribution to fluctuations of the polarization function.

We obtain

$$\langle \Pi_r(\mathbf{r}_1, \mathbf{r}_3) \Pi_r(\mathbf{r}_2, \mathbf{r}_4) \rangle = -\frac{(12/\beta)}{2\pi^2} \text{Re} \int_0^\infty \epsilon d\epsilon D_\epsilon(\mathbf{r}_1, \mathbf{r}_2) \times D_\epsilon(\mathbf{r}_2, \mathbf{r}_3) D_\epsilon(\mathbf{r}_3, \mathbf{r}_4) D_\epsilon(\mathbf{r}_4, \mathbf{r}_1). \quad (56)$$

Here  $12/\beta$  is a combinatorial factor (a number of 4-diffusion diagrams); the function  $D_\epsilon$  is given by the following expression,

$$D_\epsilon(\mathbf{r}, \mathbf{r}') = -\frac{1}{i\epsilon V} + \pi\nu \Pi_D(\mathbf{r}, \mathbf{r}'), \quad \Delta \lesssim \epsilon \lesssim E_c, \quad (57)$$

and decreases for  $\epsilon \geq E_c$ . If the integral over  $\epsilon$  in Eq. (56) diverges, the cut-off at energies of order of the mean level spacing  $\Delta$ , where the perturbative expression (56) ceases to be valid, should be introduced.

A naive estimate suggests that the leading contribution to the fluctuations of the polarizability as well as of the capacitance can be found by substituting the zero-mode contribution (the first term in rhs of Eq. (57)) for all the four functions  $D_\epsilon$ . However, since the integrals of the potentials  $\Phi_0$  and  $\tilde{\Phi}_0$  are zero, this term vanishes. For typical geometries the main contribution is given by the term where two of the functions  $D_\epsilon$  are replaced by the zero-mode result, while two others are represented by the diffusion propagator (the second term in rhs of Eq. (57)). Cutting off the logarithmically divergent integral at  $\Delta$  from below and at the Thouless energy  $E_c$  from above, we obtain

$$\langle \alpha_r^2 \rangle = \frac{12e^4}{\beta E^4} \nu^2 \ln g \times \left[ \frac{2}{V} \int d\mathbf{r}_1 d\mathbf{r}_2 \Phi_0(\mathbf{r}_1) \Pi_D(\mathbf{r}_1, \mathbf{r}_2) \Phi_0(\mathbf{r}_2) \right]^2 \quad (58)$$

and

$$\left\langle \left(\frac{e^2}{C}\right)_r^2 \right\rangle = \frac{12}{\beta} \nu^2 \ln g \times \left[ \frac{2}{V} \int d\mathbf{r}_1 d\mathbf{r}_2 \tilde{\Phi}_0(\mathbf{r}_1) \Pi_D(\mathbf{r}_1, \mathbf{r}_2) \tilde{\Phi}_0(\mathbf{r}_2) \right]^2. \quad (59)$$

Comparing these results with the expression for the WL corrections, Eqs. (30) and (51), we obtain a general formula relating WL corrections (calculated within GCE) to the mesoscopic fluctuations of the same quantity,

$$\text{r.m.s}(\alpha) = (3\beta \ln g)^{1/2} |\alpha_1| \quad (60)$$

and

$$\text{r.m.s}\left(\frac{e^2}{C}\right) = (3\beta \ln g)^{1/2} \left| \left( \frac{e^2}{C} \right)_1 \right|. \quad (61)$$

Note that in contrast to the WL corrections, the mesoscopic fluctuations are not expected to be sensitive to the GCE/CE difference, since the integral (56) is determined equally by all energies  $\Delta \lesssim \epsilon \lesssim E_c$ .

For particular geometries we obtain

$$\text{r.m.s} \alpha = \left( \frac{\ln g}{\beta} \right)^{1/2} \begin{cases} 4.71 l^{-1} (p_F \kappa)^{-2}, & 3\text{D} \\ 5.30 R^2 (\kappa p_F l)^{-1}, & 2\text{D} \end{cases}$$

and

$$\text{r.m.s} \frac{e^2}{C} = \left( \frac{\ln g}{\beta} \right)^{1/2} \begin{cases} 2.29 \tau^{-1} (p_F R)^{-4}, & 3\text{D} \\ 0.017 \tau^{-1} (p_F R)^{-2}, & 2\text{D} \end{cases}.$$

The remaining terms in Eq. (56), with all the four functions  $D_\epsilon$  replaced by diffusion propagator, can also be easily estimated. Their contribution is of the same order as Eqs. (60), (61), but without the logarithmic factor in the numerator. Thus, for the r.m.s. of the charging energy in addition to the term of order  $\sim \Delta (\ln g)^{1/2}/g$  (two zero modes) we obtain a correction of order  $\sim \Delta/g$  (no zero modes).

Amazingly, the relations between the weak localization correction and the amplitude of mesoscopic fluctuations, Eqs. (60) and (61), have a universal form, the same for 2D and 3D systems. We should note, however, that these results are not applicable for the case of polarizability fluctuations of a quasi-two-dimensional sample of the thickness  $h \gg l$  and area  $S \gg h^2$  in the transverse field (see Sec.II B) calculated previously by Berkovits and Altshuler [14]. In this case, the contribution from the terms with two zero-modes, Eq. (58), which can be easily calculated with the use of Eq. (36), yields the r.m.s value of the polarizability fluctuations,

$$\text{r.m.s}(\alpha^{(2)}) = \frac{(3\beta^{-1} \ln g)^{1/2} h}{\pi g \kappa^2},$$

where  $g = 2\pi\nu D h$ . On the other hand, the contribution where all the four functions  $D_\epsilon$  are replaced by the diffusion propagators is [14]

$$\text{r.m.s}(\alpha^{(4)}) = \frac{8\beta^{-1} \pi^{3/2} S^{1/2}}{g \kappa^2},$$

i.e it is larger by a factor  $\sim S^{1/2}/[h(\ln g)^{1/2}]$ , and for this particular geometry represents the leading contribution to the fluctuations of the polarizability.

## V. CONCLUSIONS

In this paper, we have calculated the weak localization (WL) correction to the polarizability and the capacitance

of a disordered sample and the mesoscopic fluctuations of these quantities. The WL corrections originate from the  $G_R G_A$  term in the polarization function, which depends on the presence or absence of the time-reversal symmetry. A change of the polarization function influences the screening and, consequently, the polarizability and the capacitance. We find that in the grand canonical ensemble, switching on the magnetic field leads to a positive correction to the polarizability and negative one to the capacitance. In the canonical ensemble the magnitude of the effect is the same (up to a numerical coefficient  $\sim 2.75$ ), however the sign is reversed.

Calculating the mesoscopic fluctuations of the polarizability, we find that for typical geometries they are related to the value  $\alpha_1$  of the WL correction as follows (see Eq. (60)):

$$\text{r.m.s.}(\alpha) = (3\beta \ln g)^{1/2} |\alpha_1|.$$

The same conclusion is valid for the capacitance, see Eq.(61). Therefore, the magnitude of fluctuations exceeds considerably the value of the WL correction. This should be contrasted with the relation of the corresponding quantities for the case of the conductance:

$$\begin{aligned} \text{r.m.s.}(g) &\sim 1; \\ |g_1| &\sim \begin{cases} 1, & \text{quasi-1D} \\ \ln(L/l), & 2\text{D} \\ L/l, & 3\text{D} \end{cases} \end{aligned}$$

so that  $|g_1| \gg \text{r.m.s.}(g)$  in 2D and 3D and  $|g_1| \sim \text{r.m.s.}(g)$  in the quasi-1D geometry. As our results show, an experimental observation of the magnetopolarizability of mesoscopic samples requires an experimental setup with large number of such samples, which would reduce the fluctuations.

Mesoscopic fluctuations of the charging energy contribute to the fluctuations of the conductance peak spacings in the addition spectra of quantum dots in the Coulomb blockade regime [15–18]. However, as follows from Eqs.(53), (61), the magnitude of these fluctuations is much smaller (by a factor  $\sim (\ln g)^{1/2}/g$ ) than the level spacing  $\Delta$ . Therefore, the contribution of the charging energy fluctuations to the fluctuations of the peak spacings is parametrically smaller than the effect of electron level fluctuations, which is given by the random matrix theory and is of order of  $\Delta$ . The charging energy fluctuations represent one (but not the only one) of the contributions to the enhancement of the peak spacing fluctuations as compared to the random matrix theory. This problem was considered in detail in Ref. [20].

Finally, we would like to mention once more that we assumed the screening length to be much larger than the wave length, or in other words, that  $r_s \ll 1$ , where  $r_s = e^2/\epsilon v_F$ . In the opposite case,  $r_s > 1$  (but still below the Wigner crystallization threshold), one can roughly estimate the result assuming the screening length to be approximately given by the distance between electrons.

This leads to an enhancement of the above results for the WL correction and the r.m.s. amplitude of fluctuations by a factor of order of  $r_s$  (resp.  $r_s^2$ ) for the polarizability and capacitance, respectively.

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## APPENDIX A. MAGNETOPOLARIZABILITY IN THE CANONICAL ENSEMBLE

In this Appendix we calculate the magnetopolarizability  $\alpha_B$  defined by Eq. (35) in the canonical ensemble. For this purpose we first rewrite our derivation of the Sec. II in terms of exact eigenfunctions and energy eigenvalues, in the same manner it was done originally by GE [4], and later by NRB [13]. Using the results, derived for the correlation of eigenfunctions in Refs. [6,29] (unitary ensemble) and Appendix B (orthogonal ensemble), we first obtain the weak localization correction in the grand canonical ensemble. It is in full agreement with the results obtained in Sec. II. Then we generalize the derivation to the CE case.

Our conclusion is that the CE magnetopolarizability,  $\alpha_B^{CE}$ , can be easily obtained from the GCE value,  $\alpha_B^{GCE}$ , as follows:

$$\alpha_B^{CE} = -A_{CE}\alpha_B^{GCE}, \quad A_{CE} = 2.75. \quad (62)$$

We should stress here that the derivation given below uses only the linear response formalism and the properties of the eigenfunction and eigenvalue statistics in disordered systems. It relies on the fact that the WL correction to the polarizability is determined by the energy range where the level correlation is important. It can be repeated for the case of the capacitance, where one obtains a relation analogous to Eq. (62). In contrast to this, the fluctuations of both quantities are determined by the energies  $\Delta \ll \epsilon \ll E_c$ , where the correlation of levels does not play any role, and the difference between CE and GCE values is not expected.

### Magnetopolarizability in GCE: derivation à la Gor'kov and Eliashberg

We start from Eq. (23), which can be rewritten in terms of the exact single-particle states as follows,

$$\alpha_B(\omega) = \frac{2e^2}{E^2} \int d\mathbf{r}_1 d\mathbf{r}_2 \Phi_0(\mathbf{r}_1) \delta \Pi(\mathbf{r}_1, \mathbf{r}_2) \Phi_0(\mathbf{r}_2). \quad (63)$$

Here we introduced the symbol  $\delta$  denoting the difference between the quantities in unitary and orthogonal ensembles,

$$\delta(\dots) = (\dots)_{GUE} - (\dots)_{GOE},$$

and the polarization function  $\Pi$  is expressed as follows,

$$\begin{aligned} \Pi(\mathbf{r}_1, \mathbf{r}_2) &= \sum_{m \neq n} \psi_m^*(\mathbf{r}_1) \psi_n(\mathbf{r}_1) \psi_n^*(\mathbf{r}_2) \psi_m(\mathbf{r}_2) \\ &\times \frac{n_F(\epsilon_m) - n_F(\epsilon_n)}{\omega - \epsilon_m + \epsilon_n + i0}, \end{aligned} \quad (64)$$

$m$  and  $n$  being the exact single-particle states. Thus, for  $\omega \ll \Delta$  we obtain an expression valid both in GCE and CE,

$$\alpha_B(\omega) = \frac{4e^2}{E^2} \delta \left\langle \sum_{\epsilon_n < \epsilon_F < \epsilon_m} \frac{1}{\epsilon_m - \epsilon_n} |(\Phi_0)_{mn}|^2 \right\rangle. \quad (65)$$

In GCE the position of the Fermi level can be arbitrary, and we replace the sum in Eq. (65) by an integral with the level correlation function  $R_2(\epsilon)$ :

$$\sum_{\epsilon_n < \epsilon_F < \epsilon_m} (\dots) = \Delta^{-2} \int_0^\infty \epsilon d\epsilon R_2(\epsilon) (\dots). \quad (66)$$

Using the sum rule for the eigenfunctions,

$$\left\langle \sum_n \psi_m^*(\mathbf{r}_1) \psi_n(\mathbf{r}_1) \psi_n^*(\mathbf{r}_2) \psi_m(\mathbf{r}_2) \right\rangle = V^{-1} \delta(\mathbf{r}_1 - \mathbf{r}_2),$$

we obtain for the magnetopolarizability

$$\alpha_B = -\frac{2e^2}{E^2 \Delta} \delta \left\langle |(\Phi_0)_{mm}|^2 \right\rangle. \quad (67)$$

The above derivation of Eq. (67) is essentially equivalent to that of Refs. [4,13]. Now, using Eq. (79) for the case of the orthogonal symmetry and Ref. [29] for the unitary one, we write ( $r = |\mathbf{r}_1 - \mathbf{r}_2|$ )

$$\begin{aligned} &V^2 \langle |\psi_m(\mathbf{r}_1) \psi_m(\mathbf{r}_2)|^2 \rangle_\epsilon \\ &= \begin{cases} [1 + 2\Pi_D(\mathbf{r}_1, \mathbf{r}_2)] [1 + 2k_d(r)], & GOE \\ [1 + \Pi_D(\mathbf{r}_1, \mathbf{r}_2)] [1 + k_d(r)], & GUE \end{cases} \end{aligned}$$

Separating the dominating diffusion term, we obtain for the magnetopolarizability in the grand canonical ensemble

$$\alpha_B = \frac{2e^2}{V^2 E^2 \Delta} \int d\mathbf{r}_1 d\mathbf{r}_2 \Phi_0(\mathbf{r}_1) \Pi_D(\mathbf{r}_1, \mathbf{r}_2) \Phi_0(\mathbf{r}_2), \quad (68)$$

which coincides with Eq. (30).

## Canonical ensemble

To realize the canonical ensemble, we apply the method previously developed in Refs. [30,31]. Namely, we fix the number of electrons to be integer in each individual sample, but allow it to fluctuate slightly from sample to sample. This type of ensemble is realized by pinning the Fermi-level to one of the single-particle levels  $\epsilon_k$ :  $\epsilon_F = \epsilon_k + 0$ .

Now instead of Eq. (66), one should split the sum over energy levels in Eq. (65) into two. The first contribution consists of the terms with  $m = k$ , and can be transformed to the integral with the use of the two-level correlation function  $R_2$ . The rest of the sum requires the three-level correlator  $R_3(0, \epsilon, \epsilon_1)$ , which corresponds to the probability to find three levels with energies 0,  $-\epsilon_1$ , and  $\epsilon - \epsilon_1$  (counted from the Fermi-surface). Thus, we obtain

$$\sum_{\epsilon_n < \epsilon_F < \epsilon_m} (\dots) = \Delta^{-1} \int_0^\infty d\epsilon R_2(\epsilon)(\dots) \quad (69)$$

$$+ \Delta^{-2} \int_0^\infty d\epsilon \int_0^\epsilon d\epsilon_1 R_3(0, \epsilon_1, \epsilon)(\dots),$$

and for the magnetopolarizability

$$\alpha_B = \frac{4e^2}{E^2 \Delta^2} \delta \left\{ \int_0^\infty \frac{d\epsilon}{\epsilon} \right. \quad (70)$$

$$\left. \times \left[ R_2(\epsilon) \Delta + \int_0^\epsilon d\epsilon_1 R_3(0, \epsilon_1, \epsilon) \right] \left\langle |\Phi_0|_{mn}^2 \right\rangle_\epsilon \right\}.$$

Since the integral over  $\epsilon$  converges for  $\epsilon \sim \omega$ , we can replace the matrix element by its low-frequency limit. For  $\epsilon \ll E_c$  we obtain for the correlation of the wavefunctions with  $m \neq n$  (see Eq. (82) for the orthogonal symmetry and Ref. [29] for the unitary one):

$$V^2 \langle \psi_m^*(\mathbf{r}_1) \psi_n(\mathbf{r}_1) \psi_m(\mathbf{r}_2) \psi_n^*(\mathbf{r}_2) \rangle_{\epsilon_0, \epsilon}$$

$$= \begin{cases} k_d(r) + [1 + k_d(r)] \Pi_D(\mathbf{r}_1, \mathbf{r}_2), & GOE \\ k_d(r) + \Pi_D(\mathbf{r}_1, \mathbf{r}_2), & GUE \end{cases}.$$

In the cases when the diffusion dominates [32], one can replace this eigenfunction correlator for either symmetry by  $\Pi_D$ . As a result, taking into account Eq. (68), we obtain Eq. (62), with the coefficient  $A_{CE}$  expressed in terms of the level correlation functions,

$$A_{CE} = -1 - 2 \int_0^\infty \frac{ds}{s} \quad (71)$$

$$\times \delta \left[ R_2(s) + \int_0^s ds_1 (R_3(0, s_1, s) - R_2(s)) \right],$$

where we have made a change of variables  $s = \pi\epsilon/\Delta$ . Note that Eq. (71) differs from the similar expression derived by NRB [13] by the second term in the rhs.

In the leading approximation the correlation function  $R_2$  and  $R_3$  may be taken from the random matrix theory

[33]. The first integral in the rhs of Eq. (71) can be calculated analytically and is given by

$$I_1 = \int_0^\infty \frac{ds}{s} \delta R_2(s) = \int_0^\infty \frac{ds}{s} g(s) h(s) = \frac{1}{4} - \frac{\pi^2}{16}$$

$$\simeq -0.367,$$

in contrast to the statement of NRB that it is equal to  $-1/2$ . We have defined the functions

$$f(s) = \frac{\sin s}{s}; \quad g(s) = \frac{d}{ds} f(s); \quad h(s) = \int_s^\infty f(s_1) ds_1.$$

The second term in the rhs of Eq. (71) after a lengthy algebra can be expressed as follows:

$$I_2 = \int_0^\infty \frac{ds}{s} \int_0^s ds_1 \delta (R_3(0, s_1, s) - R_2(s))$$

$$= 2 \int_0^\infty \frac{ds}{s} \int_0^s ds_1 \{ g(s_1) h(s_1) - f(s) g(s_1) h(s - s_1) \\ + g(s) f(s_1) h(s - s_1) + h(s) f(s_1) g(s - s_1) \}.$$

Calculating this numerically, we find  $I_2 = -1.509$  and thus,  $A_{CE} = -1 - 2(I_1 + I_2) = 2.753$ .

Note that in the above derivation we neglected the contribution of the so-called Debye processes (relaxation to the instantaneous equilibrium distribution due to coupling with phonons or other possible inelastic processes) [34,35,13]. These processes do not exist in a closed sample in the limit of zero temperature ( $T \ll \Delta$ ) that we are considering [36].

## APPENDIX B. CORRELATIONS OF EIGENFUNCTIONS IN DISORDERED SYSTEMS: ORTHOGONAL ENSEMBLE

In this Appendix we derive the expressions for the correlations of the eigenfunctions in the orthogonal ensemble in the same way as these were obtained in Ref. [29] for the unitary ensemble. We restrict ourselves to the terms of order  $g^{-1}$ .

Following [29] we define the eigenfunctions correlators (see Eq. (2)),

$$\eta(\mathbf{r}_1, \mathbf{r}_2, \epsilon) = \langle |\psi_k(\mathbf{r}_1) \psi_k(\mathbf{r}_2)|^2 \rangle_\epsilon$$

$$\equiv \frac{\langle \sum_k |\psi_k(\mathbf{r}_1) \psi_k(\mathbf{r}_2)|^2 \delta(\epsilon - \epsilon_k) \rangle}{\langle \sum_k \delta(\epsilon - \epsilon_k) \rangle}, \quad (72)$$

$$\beta(\mathbf{r}_1, \mathbf{r}_2, \epsilon, \omega) = \langle |\psi_k(\mathbf{r}_1) \psi_l(\mathbf{r}_2)|^2 \rangle_{\epsilon, \omega}, \quad k \neq l,$$

and

$$\gamma(\mathbf{r}_1, \mathbf{r}_2, \epsilon, \omega) = \langle \psi_k^*(\mathbf{r}_1) \psi_l(\mathbf{r}_1) \psi_k(\mathbf{r}_2) \psi_l^*(\mathbf{r}_2) \rangle_{\epsilon, \omega}, \quad k \neq l.$$

The quantities  $\eta$  and  $\beta$  are related as follows,

$$B \equiv \eta(\mathbf{r}_1, \mathbf{r}_2, \epsilon) \Delta^{-1} \delta(\omega) + \beta(\mathbf{r}_1, \mathbf{r}_2, \epsilon, \omega) \Delta^{-2} R_2(\omega)$$

$$= \nu^2 + (2\pi^2)^{-1} \text{Re} \{ \langle G^R(\mathbf{r}_1, \mathbf{r}_1, \epsilon) G^A(\mathbf{r}_2, \mathbf{r}_2, \epsilon + \omega) \rangle \\ - \langle G^R(\mathbf{r}_1, \mathbf{r}_1, \epsilon) \rangle \langle G^A(\mathbf{r}_2, \mathbf{r}_2, \epsilon + \omega) \rangle \}; \quad (73)$$

here the two-level correlation function,

$$R_2(\omega) = \Delta^2 \left\langle \sum_{k \neq l} \delta(\epsilon - \epsilon_k) \delta(\epsilon + \omega - \epsilon_l) \right\rangle, \quad (74)$$

is introduced.

The right-hand side of the expression (73) can be directly calculated with the use of the supersymmetry technique. For the case of preserved time-reversal symmetry (orthogonal ensemble) one obtains

$$\begin{aligned} B(\mathbf{r}_1, \mathbf{r}_2, \epsilon, \omega) = & -(2\pi^2)^{-1} \text{Re} \left\{ \langle g_{b1,b1}^{11}(\mathbf{r}_1, \mathbf{r}_1) \right. \\ & \times g_{b1,b1}^{22}(\mathbf{r}_2, \mathbf{r}_2) + g_{b1,b1}^{12}(\mathbf{r}_1, \mathbf{r}_2) g_{b1,b1}^{21}(\mathbf{r}_2, \mathbf{r}_1) \rangle_F \\ & \left. - \langle g_{b1,b1}^{11}(\mathbf{r}_1, \mathbf{r}_1) \rangle_F \langle g_{b1,b1}^{22}(\mathbf{r}_2, \mathbf{r}_2) \rangle_F \right\}. \end{aligned} \quad (75)$$

Here  $\langle \dots \rangle_F$  denotes the averaging with the action of the supermatrix sigma-model  $F[Q]$ :

$$\begin{aligned} \langle \dots \rangle_F &= \int DQ(\dots) \exp(-F[Q]), \\ F[Q] &= -\frac{\pi\nu}{8} \int d\mathbf{r} \text{Str}[D(\nabla Q)^2 + 2i(\omega + i0)\Lambda Q], \end{aligned} \quad (76)$$

where  $D$  is the diffusion coefficient,  $Q = T^{-1}\Lambda T$  is a  $8 \times 8$  supermatrix,  $\Lambda = \text{diag}(1, 1, 1, 1, -1, -1, -1, -1)$ , and  $T$  belongs to the supercoset space  $U(2, 2|4)/U(2|2) \times U(2|2)$ . The symbol  $\text{Str}$  denotes the supertrace (trace over bosonic degrees of freedom minus that over fermionic ones). The upper matrix indices correspond to the retarded-advanced decomposition, while the lower indices denote the boson-fermion one (here we need only the index  $b1$ , which denotes one of two bosonic components of a supervector). The Green's function  $g$  in Eq. (75) is the solution to the matrix equation:

$$\begin{aligned} & \left[ -i\left(\epsilon + \frac{\omega}{2} - \hat{H}_0\right) - \frac{i}{2}(\omega + i0)\Lambda + Q/2\tau \right] g(\mathbf{r}, \mathbf{r}') \\ &= \delta(\mathbf{r} - \mathbf{r}'). \end{aligned} \quad (77)$$

Expressing these functions through the matrices  $Q$  and taking into account Eq. (73), we arrive at the following equation valid in for an arbitrary diffusive system:

$$\begin{aligned} & 2\pi^2 \left[ \frac{\eta(\mathbf{r}_1, \mathbf{r}_2, \epsilon)}{\Delta} \delta(\omega) + \frac{\beta(\mathbf{r}_1, \mathbf{r}_2, \epsilon, \omega)}{\Delta^2} R_2(\omega) \right] \\ &= (\pi\nu)^2 \text{Re} \left\{ 1 - \langle Q_{b1,b1}^{11}(\mathbf{r}_1) Q_{b1,b1}^{22}(\mathbf{r}_2) \rangle_F \right. \\ & \quad \left. - 2k_d(r) \langle Q_{b1,b1}^{12}(\mathbf{r}_1) Q_{b1,b1}^{21}(\mathbf{r}_1) \rangle_F \right\}, \end{aligned} \quad (78)$$

here the function  $k_d$  is defined in Eq. (5), and  $r = |\mathbf{r}_1 - \mathbf{r}_2|$ . The separation of the the rhs of Eq. (78) into the singular (proportional to  $\delta(\omega)$ ) and regular parts allows one to obtain the quantities  $\alpha(\mathbf{r}_1, \mathbf{r}_2)$  and  $\beta(\mathbf{r}_1, \mathbf{r}_2, \omega)$ .

For the case of a metallic system in the weak localization regime, the sigma-model correlation functions  $\langle Q_{b1,b1}^{11}(\mathbf{r}_1) Q_{b1,b1}^{22}(\mathbf{r}_2) \rangle_F$  and  $\langle Q_{b1,b1}^{12}(\mathbf{r}_1) Q_{b1,b1}^{21}(\mathbf{r}_2) \rangle_F$  can be calculated for relatively low frequencies  $\omega \ll E_c$  with

the use of a general method developed in Refs. [37,38] which allows one to take into account spatial variations of the field  $Q$ . The results are obtained in the form of an expansion in  $g^{-1}$ . Up to the terms of order  $g^{-1}$ , we obtain

$$\begin{aligned} & \langle Q_{b1,b1}^{11}(\mathbf{r}_1) Q_{b1,b1}^{22}(\mathbf{r}_2) \rangle_F \\ &= 1 - 2\tilde{R}(\omega) - \frac{4i\Delta}{\pi(\omega + i0)} \Pi_D(\mathbf{r}_1, \mathbf{r}_2) \end{aligned}$$

and

$$\begin{aligned} & \langle Q_{b1,b1}^{12}(\mathbf{r}_1) Q_{b1,b1}^{21}(\mathbf{r}_2) \rangle_F = -2 \frac{i\Delta}{\pi(\omega + i0)} \\ & - 2 \left( \tilde{R}(\omega) + \frac{i\Delta}{\pi(\omega + i0)} \right) \Pi_D(\mathbf{r}_1, \mathbf{r}_2). \end{aligned}$$

Here the diffusion propagator  $\Pi_D$  is defined by Eq. (3), and we have introduced the function  $\tilde{R}(\omega) = [1 + S(\omega)]/2$ , where  $S(\omega)$  is given by Eq. (16). Note that the two-level correlation function,  $R_2(\omega)$ , is the real part of  $\tilde{R}(\omega)$ .

Now, separating regular and singular parts in rhs of Eq. (78), we obtain the following result for the autocorrelations of the same eigenfunction,

$$\begin{aligned} & V^2 \langle |\psi_k(\mathbf{r}_1) \psi_k(\mathbf{r}_2)|^2 \rangle_\epsilon \\ &= [1 + 2k_d(r)] [1 + 2\Pi_D(\mathbf{r}_1, \mathbf{r}_2)], \end{aligned} \quad (79)$$

and for the correlation of amplitudes of two different eigenfunctions ( $k \neq l$ )

$$V^2 \langle |\psi_k(\mathbf{r}_1) \psi_l(\mathbf{r}_2)|^2 \rangle_{\epsilon, \omega} - 1 = 2k_d(r) \Pi_D(\mathbf{r}_1, \mathbf{r}_2). \quad (80)$$

The result (79) for  $\mathbf{r}_1 = \mathbf{r}_2$  is the inverse participation ratio previously obtained in Ref. [38], while that for an arbitrary spatial separation was found in the zero-mode approximation ( $g = \infty$ ) in Ref. [39].

Now we turn to the correlation function  $\gamma$ . Similarly to Ref. [29], we obtain a relation

$$\begin{aligned} & 2\pi^2 \left[ \frac{\eta(\mathbf{r}_1, \mathbf{r}_2, \epsilon)}{\Delta} \delta(\omega) + \frac{\gamma(\mathbf{r}_1, \mathbf{r}_2, \epsilon, \omega)}{\Delta^2} R_2(\omega) \right] \\ &= -(\pi\nu)^2 \text{Re} \left\{ \langle Q_{b1,b1}^{12}(\mathbf{r}_1) Q_{b1,b1}^{21}(\mathbf{r}_2) \rangle_F + k_d(r) \right. \\ & \quad \left. \times [\langle Q_{bb}^{11}(\mathbf{r}_1) Q_{bb}^{22}(\mathbf{r}_1) \rangle_F + \langle Q_{b1,b1}^{12}(\mathbf{r}_1) Q_{b1,b1}^{21}(\mathbf{r}_2) \rangle_F - 1] \right\}. \end{aligned} \quad (81)$$

Separating again the rhs into the regular and singular parts, we recover Eq. (79) and obtain

$$\begin{aligned} & V^2 \langle \psi_k^*(\mathbf{r}_1) \psi_l(\mathbf{r}_1) \psi_k(\mathbf{r}_2) \psi_l^*(\mathbf{r}_2) \rangle_{\epsilon, \omega} \\ &= k_d(r) + [1 + k_d(r)] \Pi_D(\mathbf{r}_1, \mathbf{r}_2), \quad k \neq l. \end{aligned} \quad (82)$$

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